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Asset Pricing Puzzles: A Comparison of India and Pakistan

Mohammad Azam *

Abstract: *This study spotlights the price fluctuations between risk-factors and portfolio returns which motivates the study to scrutinize multifactor asset pricing models in India and Pakistan equity markets during January 2003 to December 2022. The study sample included monthly data from 250 non-financial enrolled enterprises in each market. The data was evaluated using the time-series Ordinary Least Square regression estimate approach. We find that market-factor performs significant contribution independently though augmenting the liquidity-factor which resultantly enhances an insignificant and tenuous impact in Pakistan equity market while investment-factor shows insignificant contribution independently though augmenting the liquidity-factor which resultantly enhances the impact positively in Indian equity market. Moreover, the value-factor is not redundant for both the markets. However, based on absolute mean alpha test, the four-factor model for Indian equity market however, seven-factor model for Pakistani equity market outperform other asset pricing models.*

Keywords: Asset Pricing Puzzles, Time-series Ordinary Least Square regression, Gibbons, Ross & Shanken (1989) test, Bombay Stock Exchange, Pakistan Stock Exchange.

Background of the Study and Rationale

Stock market investors are looking for secure investment avenues (Kumar, Kumar, & Singh, 2023), hence they are hyperconscious in the intrinsic value of a stock due to a risk-averse approach and a lack of vast resources to spend in the stock market. To remain buoyant, they examine the stock utilizing a risk-premium estimation practice to obtain a decision and directly answer an intriguing question straightforwardly that which stock to be included in the portfolio. As a result, the mystery of stock pricing riddles is a complicated phenomenon that is essential for understanding rational investment decisions in capital markets through the adoption of APMs estimation techniques.

Since researchers and academics established numerous theories and empirical techniques for forecasting expected returns in the form of sophisticated asset pricing models (henceforth APMs), the quandary has remained a complex phenomenon that has sparked a debate that needs to be thoroughly investigated. The origins of APMs may be traced back to 1930s, when (Graham & Dodd, 1934) led the first inquiry into this riddle. Their statement that every stock has an intrinsic value and that investing professionals seek to

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purchase stock at a discount to its intrinsic value in order to maximize excess returns pioneered a new finance discipline to investigate the factors that explain a stock's intrinsic value. Portfolio managers and analysts, on the other hand, anticipate larger excess returns in order to outperform the market by mitigating risks through diversification. Sharpe (1964); Lintner (1975); Mossin (1966) later pioneered the Capital Asset Pricing Model-Single-factor model (hereafter 1FM), which deals with macro-determinants of explaining stock or portfolio returns using market excess returns. APMs, on the other hand, are used to decide which proxy or anomaly is most suited to explaining the return of a portfolio or stock in order to calculate the required rate of return.

These models employ financial statements data as micro-determinants to evaluate a stock or portfolio based on firm-characteristics such as the price-to-earnings (P—E) ratio and book-to-market (B—M) ratio (Banz, 1981). Later on, Fama and French (1993) divert the discipline into another track by introducing three-factor model (henceforth 3FM) using size and value determinants with 1FM. Alternatively, Carhart (1997) added momentum-factor (henceforth 4FM) which also contributed significant results in explaining stock returns. Bhatnagar and Ramlogan (2010) argued that it substantially improves the R-square ranges from 75-94% to 89-96%. In between, Liu (2006) initiated another factor named multidimensional liquidity which produced enriched explanatory power to the model known as two-factor model (henceforth 2FM). Furthermore, Fama and French (2015) stepped up alternative model by adding profitability and investment factors known as five-factor model (henceforth 5FM). Prior studies have shown statistically significant contribution of the model around the globe including developed and emerging equity markets. Conclusively, there are several theoretical foundations for the anomalies, which are supported by empirical studies employing established, developing, and frontier equities markets which enriched the APMs discipline which evidenced significant findings but few markets showed insignificant findings for the factors while several models showed invalid results in numerous equity markets which motivates the author to comparatively study both the markets using various APMs.

A plethora of studies conducted exclusively on Indian and Pakistani equity markets in the realm of APMs which are widely reported and extensively explored in the literature whereas comparatively there are very limited studies investigated the sophisticated APMs particularly liquidity and momentum augmented APMs. On the other hand, studies have investigated various liquidity proxies such as turnover, Amihud and ILLIQ though they used one dimensional liquidity models on the other hand, comparing both the equity markets used numerous single and multiple APMs but both momentum and multidimensional liquidity augmented with 5FM is not observed by the author. Therefore, this study fills the gaps by using seven various APMs including 7FM to investigate both the markets by different perspectives and contribute to the body of knowledge. Moreover, a large amount of studies has demonstrated a strong and consistent link between liquidity and stock returns around the globe but limited work has been observed using (Liu, 2006) multidimensional liquidity link with stock returns in Asian equity markets which needs to be augmented with nested APMs for further robust findings. This study attempts to fill the gap which risk-factors explain stock prices fluctuations and support empirically evidences for investors and portfolio managers to be considered before

making decisions in investing both the markets. Fama and French (1992) used cross-sectional two-steps regression though (Fama & French, 1993) used time-series OLS regression technique which they follow for more studies and observed more appropriate for the APMs. Therefore, this study also uses the time-series OLS regression approach.

Economic Overview (India and Pakistan)

India is currently ranked as the fifth-largest economy in terms of nominal gross-domestic product (GDP) and third-largest by purchasing power parity (Nandy, 2023). Over the past two decades, the country has experienced average annual economic growth of 6-7%, earning it the distinction of being the second-fastest-growing major economy in the world. India is classified as a newly industrialized country, and is a member of the G-20 major economies, and a developing nation with an average growth rate of about 7% in the last two decades. It has the third-largest active workforce in the world, and is projected to become the second-largest consumer market by 2025. The service sector is the largest contributor to the Indian economy, accounting for about 53.2% of the country's GDP in 2019. Other major economic contributors are the industrial and agricultural sectors, which accounted for 28.6% and 18.2% of the GDP respectively (Thomas, 2023). India is one of the world's leading producers of agricultural products and is the world's second-largest producer of rice and wheat. India is also one of the world's largest producers of spices, fruits, and vegetables. The manufacturing sector is the second-largest contributor to the Indian economy, and is mainly driven by the automobile (Malik, 2023).

The oldest equity market in Asia is the Bombay Stock Exchange (BSE-500 Index), which was founded in 1875 (Elangovan, Irudayasamy, & Parayitam, 2022). It includes all 20 of the main economic sectors. Additionally, it is ranked as the world's tenth-oldest equities market. Based on market capitalization, it is likewise regarded as one of the biggest markets in the world. By joining in 2012, it reached the milestone of becoming a Partner Exchange of the United Nations Sustainable Equity Market project. In October 2018, it launched commodities futures contracts in gold and silver, becoming the first stock exchange in the nation to do so. With a population of over 210 million (the world's 5th-largest), Pakistan has the 23rd largest economy in the world based on purchasing power parity (Ali, Maryam, Saddique, & Ikram, 2023). In 2019, this equated to a nominal GDP per capita of $1,357, ranking 154th globally, and a PPP GDP per capita of 5,839, placing Pakistan 132nd worldwide. Unfortunately, the nation has endured various difficulties in the past, including internal political disputes, a rapidly-growing population, mixed foreign investment, and an expensive, ongoing conflict with India. However, since the end of 2016, the economy has improved significantly. In 2018, Pakistan achieved her highest-ever growth rate of 5.79%. By 2020, it was estimated that the country's GDP growth rate would reach 6.2%. The service sector accounts for about 54% of GDP, followed by the industrial sector at 34.6%, and the agriculture sector at 11.4% (Jabeen & Khan, 2022). Major industries include textiles, fertilizers, cement, oil refineries, dairy products, food processing, beverages, construction materials, clothing, paper products and shrimp.

In comparison, Pakistan and India are two of the most populous countries in the world. Although both countries have similar over-population sizes, their economic per-

formance differs significantly. In comparison to India, Pakistan has a much smaller GDP, with India's GDP being 5.2 times larger. India is also significantly more industrialized and has a much higher Human Development Index (HDI) than Pakistan. India's GDP per capita is almost 4 times higher than Pakistan's. In terms of infrastructure, India has a much larger network of roads, railways, ports, and airports. India also has a much more developed banking system, with over 1,000 commercial banks, compared to Pakistan's 100. Both countries face similar issues such as high levels of poverty, unemployment, and inflation, although India's rates are generally lower. Pakistan also has a higher rate of illiteracy and a weaker educational system. Overall, India has a much stronger economy than Pakistan and is better positioned to take advantage of global economic opportunities.

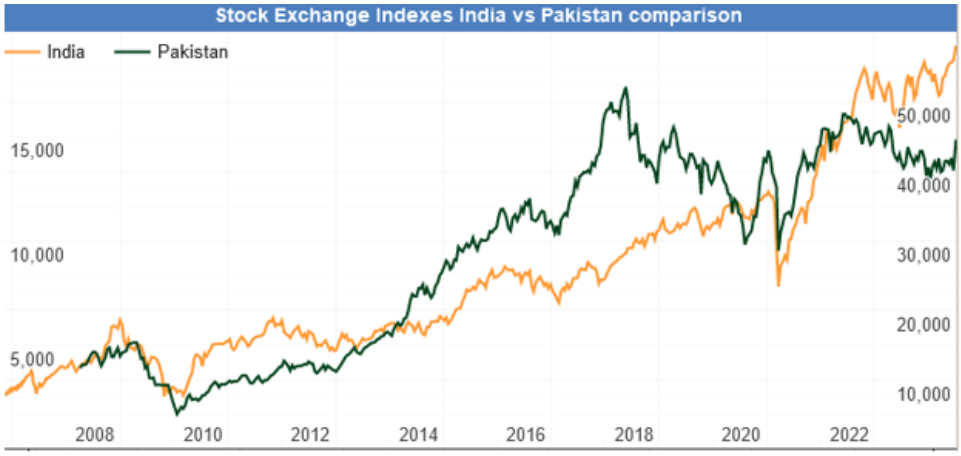
The Pakistan Stock Exchange (formerly known as the Karachi Stock Exchange) was established in 1947 and has been recognized as the best performing stock exchange in the world. It currently has 531 companies listed with a market capitalization of Rs.7.68 trillion. It was liberalized in 1991 while in 2001-02, the KSE-100 Index stood at only 1,366 points with the market-cap of Rs. 407 billion. However, at the start of 2003, the bullish trend has been observed. In October, 2007, it had settled over 14,430 points with market-cap of more than Rs. 4 trillion, depicting almost nine-time growth over the fiscal year 2001-02. The stock market had even absorbed the most devastating crashes for the retail and small investors in the years 2003, 2005 and 2008. According to the Economic Survey of Pakistan (2018-2019), 90% of the country's trading activities take place at the PSX. Over the last two decades, the war against terrorism has significantly altered the economic landscape of Pakistan, resulting in an uncertain environment for stock market investments (Rasheed, Malik, Haider, & Shakeel, 2023). It has been estimated that the country has incurred a total cost of approximately seventy billion dollars due to terrorism, which is almost half of Pakistan's total debt, as reported in the Pakistan Economic Survey (2012-2013). There is no doubt that terrorism has had a negative impact on both the stock market and the overall economy.

Since 1990, the significant developments of the KSE-100 price index, market capitalization and changes to settlement periods have increased the importance of the PSX. These changes were the result of financial liberalization, with deregulation policies implemented to open up the market to foreign investors and attract local investors (Chakraborty, 2006). Institutional progress and reforms have resulted in increased disclosure of information through regular reports, notification of dividends, official meetings and daily quotation. Measures have been taken by the PSX to protect investors from the high volatility of stock prices and streamlines the trading activities of the stock market. These initiatives have reduced the chances of fraud and delays in transfer and decreased the volatility of stock prices. The exchange also supplies real-time information to investors via the internet. The Security and Exchange Commission of Pakistan (SECP) provides guidelines to ensure good governance with the aim of boosting investor confidence by increasing transparency in listed company trading activities (Ahmed & Mustafa, 2013). Furthermore, the government has privatized many institutions to reduce organizational problems and increase financial stability, generating capital from the stock market.

The 2018-19 economic survey of Pakistan revealed that the stock market saw a remarkable development between July 2018 and March 2019. However, a variety of good

events that happened politically and economically were responsible for this growth. This includes the implementation of Capital Gain Tax Rules, demutualization of the stock, a sizable cut in the State Bank of Pakistan's (SBP) discount rate to 10% on 3rd December 2018, increased foreign investment in stock, diminishing rates of inflation, and political stability in the country. All these components fostered a flourishing capital market. Figure 1 demonstrates the both market overview to understand the market mechanisms at a glance:

Figure 1



Literature Review and Hypotheses Development

A plethora of prior research has presented empirical evidences indicating there is a significant linkage between risk and returns employing numerous APMs with a range of anomalies around the world, particularly in South-Asian subcontinents due to their strategic position and potential economic expansion. However, the discipline followed the Markowitz (1952) concept of modern portfolio theory, a major milestone which is attributed with the utilization of mean-variance analysis in portfolio selection. It provided a rigorous mathematical framework to analyze the risk-return tradeoff of a portfolio of assets and argued that investors should construct portfolios that maximize potential returns for a given level of risk. This theory served as the foundation for the development of quantitative investment strategies and has become one of the most influential and widely cited works in financial economics. Later on, the 1FM is a model used to calculate the expected return of an asset based on its level of risk. It states that the expected return of a stock is equal to the risk-free rate plus a risk premium, which is proportional to the beta of stock. The beta measures the systematic risk of stock and is calculated by comparing the stock returns to the market returns (Lee, Hooy, & Brooks, 2023). This model has been applied extensively and has been verified through empirical testing all over the world, continuing to generate

successful results due to its simplicity (Alaoui Taib & Benfeddoul, 2023; Kostin, Runge, & Adams, 2021). Conversely, 1FM validity has been questioned as poor performing model because of data abnormality.

Later on, an alternative two-factor model named the multidimensional liquidity augmented 1FM was proposed by Liu (2006) which is a modified version of the traditional two-factor APM (2FM). This model takes into account the liquidity risk premium associated with investing in illiquid assets. The model adds a second factor, the "liquidity factor", which captures the effect that liquidity has on asset returns. The liquidity factor is then added to the 1FM, resulting in a two-factor model which is more accurate in pricing assets. Moreover, there is significant nexus between financial statement and liquidity of stock such as (Majeed & Yan, 2022) explored the nexus between financial statement comparability and stock liquidity using (Corwin & Schultz, 2012) model in an oblique fashion utilizing non-financial Chinese firms for the period 2005–2018. The findings reveal that the substantial financial comparability elevates financial transparency, which in turn enhances the informational ambience and boosts stock liquidity.

After long time, the Fama and French (1992, 1993) three-factor model (3FM) as an augmented version of the nested 1FM was developed by adding size and value factors in addition to the market factor. The model is used to explain the cross-section of expected returns of different assets. The three factors are the market risk factor (the return of the broad stock market), the size factor (the return of small stocks minus the return of large stocks) and the value factor (the return of high book-to-market stocks (growth) minus the return of low book-to-market stocks (value). The model suggests that stocks with high exposure to these three factors tend to have higher returns than stocks with low exposure to these factors. The model is useful for portfolio managers as it allows them to identify stocks that may outperform the market as a whole. Moreover, the size proxy is a measure of the size of a company, such as its market capitalization or total assets, used to control for the potential effect of size on returns. It is often used in an empirical APM to explain factors such as the risk premium and the size effect. By controlling for size, the model can more accurately estimate the effects of other factors such as risk, industry, and momentum. Asness, Frazzini, Israel, Moskowitz, and Pedersen (2018) examined the size effect and argued that it has strong nexus with liquidity. Studies spotlighted that small companies are illiquid, bearing liquidity risk, hence demand higher expected premium.

The Carhart (1997) four-factor model (4FM) is an APM builds on adding a fourth factor with 3FM to explain the excess return of stocks in the market. The four factors are the market, size, value and momentum. The momentum is measured by the returns of stocks over the past 12 months. The model suggests that a portfolio of stocks that have high exposures to these four factors will produce better returns than a portfolio of stocks with low exposure. Afterwards, Fama and French (2015) developed another APM known five-factor model (5FM) which adds two additional factors, namely profitability (RMW) and investment (CMA), to the original three factors: market risk (market), size (SMB), and value (HML). The five factors are intended to explain the differences in expected returns of stocks and other assets. The model is based on the idea that returns are driven by the risk associated with each factor and the sensitivity of the asset to those risks. The model argues that investors should be compensated for taking on additional risks associated

with these factors, and therefore the expected return of an asset should be related to its factor exposures. One of the criticisms on 5FM is the redundancy of value-factor which is commonly observed premises for US equity market and similarly investment-factor for Chinese market (Guo, Zhang, Zhang, & Zhang, 2017). Besides that, momentum-factor outperform for developed equity markets while less noticeable for frontier markets.

Studies have shown that both India and Pakistan equity markets are evaluated independently using nested and augmented APMs such as 3FM, 4FM 5FM and revealed statistically significant findings such as (Azam, 2022). However, to the best of author knowledge, no study has been observed to investigate the multidimensional liquidity and momentum augmented nested APMs comparatively on India and Pakistan. Therefore, this study seeks to provide an empirical contribution to the existing body of knowledge by investigating multidimensional liquidity and momentum augmented with 5FM. As the literature lacks empirical evidence from a single study, this research fills the gap by applying liquidity and momentum augmented multifactor APMs to India and Pakistan (BSE and PSX) equity markets. The prior literature investigated the model validity particularly the redundancy of value-factor specified by Fama and French (2015) and observed by numerous studies. As Shoaib and Siddiqui (2020) examined the 3FM using China, India and Pakistan stock markets for time span from 2001 to 2017 and observed significant size effect exists in all the markets. Moreover, market factor is observed insignificant while value factor as redundant for the markets. A novel risk-factor named Tobin-q risk-factor has been investigated by Azam (2022) is observed highly significant in explaining portfolio returns in PSX. Alternatively, another study conducted on 5FM using Consumer Confidence Index (CCI) as mediating variable between risk-factors and portfolio returns on PSX revealed statistically significant findings for 5FM. The overall findings of these studies revealed valid explanation for 5FM in PSX which indicating significance model for the market though CCI shows insignificant results.

There are ample literature and empirical investigations with divergence findings using 5FM around the globe and particularly on PSX. More recently, the 5FM has been investigated by Azam (2022) and revealed significant coefficients for PSX. However, Kalim, Saeed, and Kamil (2023) observed that there is no link between firm size and profitability in PSX. On the other hand, Rao, Haque and Qamar (2022) examined the Indian and Pakistani equity markets using contrarian and momentum strategies. The findings reveal significant reversal effect for both the markets. Recently, Younus (2022) examines the Fama and French (2018) six-factor (6FM) using PSX dataset from 2000 to 2017. The momentum factor is augmented with 5FM as additional factor for further investigation which statistically qualified the t-stat criteria (1.96) for the market. At the same time, the study used eleven anomalies with alternative of operating and cash profitability for further robustness. More recently, Azam (2023) investigates the multidimensional liquidity augmented APMs using liquidity as risk-factor as well as mediating variable and observed highly statistically significant findings for liquidity factor using PSX dataset. It motivates the author to investigate liquidity and momentum augmented in both India and Pakistan stock exchange for further understanding the estimation ability of APMs in both the markets. After thoroughly examining the literature review, this study develops the following hypotheses:

H_1: The stocks having small market-cap, high B—M ratios, higher profitability, higher investment, illiquidity and winner have significant nexus with higher portfolio stock returns.

H_2: The stocks having big market-cap, low B—M ratios, lower profitability, lower investment, liquidity and loser have significant nexus with lower portfolio stock returns.

H_3: This study investigates the GRS-Wald version F-test based on the hypothesis that all alpha coefficients are equal to zero (α=0).

Data and Model Specification

This research investigates the monthly closing stock returns of 250 non-financial stocks from BSE-500 Sensex and 250 non-financial stocks from PSX. The sample period is from January 2003 to December 2022 which consists of 20 years. The monthly closing stock prices were obtained from the official websites of PSX. The data used in the calculations of profitability and investment and Treasury bills rates were taken from the State Bank of Pakistan website. Lastly, the BSE-500 Sensex data was acquired from Thomson Reuters. Fama and French (1992) employed Fama and MacBeth (1973) two-steps cross-sectional regression through (Fama & French, 1993) adopted time-series OLS regression technique which gained popularity in APMs, therefore, this study uses the same methodology using the following model specifications to examine the zero intercepts hypothesis.

Model Specification

Based on time-series OLS regression, this study employs the below mention nested and augmented asset pricing models:

Capital Asset Pricing Model (1FM):

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \epsilon_i \tag{1}$$

Where, Ri - Rf, is excess returns of portfolio, Rm - Rf, is the excess returns of market, and β_m, is the coefficient of market factor.

Liu (2006) two-factor model (2FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_1(IML) + \epsilon_i \tag{2}$$

Where, IML is the Illiquid firms returns minus Liquid firms returns called Liquidity factor and β_1 is the coefficient of liquidity factor.

Fama & French (1993) three-factor model (3FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SMB) + \beta_v(HmL) + \epsilon_i \quad (3)$$

Where, SmB is the Small Minus Big firms returns called Size factor, HmL is the High minus Low firms returns called Value factor, and β_m, β_s, and β_v are the coefficients of market, size and value factors respectively.

Carhart (1997) four-factor model (4FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SMB) + \beta_v(HmL) + \beta_w(WmL) + \epsilon_i \quad (4)$$

Where, WML is the Winner Minus Loser firms returns called Momentum factor and β_w is the coefficient of momentum factor.

Fama & French (2015) five-factor model (5FM)

$$R_i - R_f = R_f + \beta_m(R_m - R_f) + \beta_s(SMB) + \beta_v(HmL) + \beta_p(RmW) + \beta_i(CmA) + \epsilon_i \quad (5)$$

Where, RMW is the Robust Minus Weak firms returns called Profitability factor, CMA is the Conservative Minus Aggressive firms returns called Investment factor and β_p, and β_i are the coefficients of profitability and investment factors respectively.

Portfolio and Factor Construction

Following Fama and French (1993, 2015), this study constructs six equally weighted portfolios based on market-cap and book-to-market (B|M Ratio) ratios. First, we sort all stocks into two equally groups such as small and big firms, then further regrouped into three categories such as high, medium and low B|M Ratio ratios such as:

Table 1
Portfolio and Factor Construction

Portfolio Construction (Independent)			Factor Construction (Dependent)	
Market-cap	**B|M Ratio**	**Portfolios**	**Small:**	**High:**
Small (50%)	High (33%)	SH	SH+SL/2	SH+BH/2
	Medium (34%)	SM		
	Low (33%)	SL	Big:	Low:
			BH+BL/2	SL+BL/2
Big (50%)	High (33%)	BH		
	Medium (34%)	BM	SMB:	HML:
	Low (33%)	BL	Small minus Big	High minus Low

Empirical Results and Discussions

Summary Statistics and Correlation Matrix

Table 3 displays the descriptive statistics for all the right-hand side (RHS) factors used in this study for both Pakistan and Indian equity markets. In mean (returns), the results

show almost similar in terms of magnitude except size, value and profitability which shows negative for PSX while positive for BSE. Comparatively, the BSE mean and maximum values show greater than PSX. On the other hand, the correlation matrix shows the nexus between each pair of factors. The maximum value is 0.56 between HML and CMA (BSE) and -0.62 between SMB and CMA (PSX) which confirms that there is not multicollinearity issues as Jr, C, J, and E (2010) argue that the value between each pair of variables in the correlation matrix should not exceed 0.90.

Table 2
Factor Explanation

S #.	Risk-Factor	Symbol	Explanation
1	Market	RmRf	Returns of PSX-100 Index minus 3 months Risk-free rates
2	Size	SMB	Returns of small size firms minus Returns of big size firms
3	Value	HML	Returns of high B—M equity minus Returns of low B—M equity
4	Profitability	RMW	Returns of high operating profitability stocks minus Returns of low operating profitability stocks
5	Investment	CMA	Returns of high investment firms minus Returns of low investment firms
6	Liquidity	IML	Returns of Illiquid stocks minus Returns of Liquidity stocks
7	Momentum	WML	Returns of Winners stocks minus Returns of Losers stocks based on (12 months)

Table 2 shows the risk-factors used in the study with explanation how these portfolios are categorized into factors.

Regression Results

The time-series OLS regression technique was used with equation (1) and the 1FM model was analyzed to examine the relationship between portfolio returns and market excess. Based on the t-stat criteria (1.96) as suggested by Younus (2022), this study also statistically qualified this criterion and observes significant t-stat at the 5% significance level (p-value = 0.0000) for all the portfolios. These results are reported in Table 4. On the left-hand side, the results of the PSX show that all coefficients for the market risk factor were statistically significant, though the R-squared ranged from 12% to 24%. On the right-hand side, the results of the BSE showed similarly statistically significant coefficients at the 5% significance level (p-value = 0.0000) for the market risk factor and an R-squared ranging from 25% to 48%. Despite these differences, both markets demonstrate the existence of the 1FM in the markets.

Table 5 reveals the outcome of the Liu (2006) two-factor model utilizing six portfolio returns. It is evident that after the introduction of liquidity as an additional factor to the model, the market risk-factor became weaker as only three coefficients were found to be statistically significant when using the PSX dataset. On the other hand, liquidity showed highly statistically significant coefficients at the 5% significance level (p-value = 0.0000) for all models. The R-squared for BL and SL portfolios were found to range from 63% to 86%, respectively. In contrast, the coefficient findings for BSE showed a highly significant yet

Table 3
Descriptive Statistics and Correlation Matrix for India and Pakistan Data

	RmRf		SMB		HML		WML		RMW		CMA		IML	
Variable	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**
Mean	0.005623	0.006145	-0.00895	0.032621	-0.02567	0.027625	0.059335	0.040179	-0.00472	0.024651	0.019116	0.029173	0.005506	0.024131
Std. Dev.	0.075318	0.07091	0.041865	0.095313	0.042092	0.081251	0.059181	0.115329	0.032991	0.097862	0.042069	0.101704	0.055749	0.092048
Min	-0.45966	-0.28277	-0.1526	-0.18913	-0.27753	-0.22469	-0.14013	-0.24281	-0.09243	-0.29468	-0.1359	-0.24162	-0.18123	-0.29989
Max	0.235415	0.327884	0.128924	0.454928	0.103502	0.444445	0.42918	0.502999	0.160848	0.469933	0.229755	0.423383	0.127334	0.430744
Obs.	246	246	246	246	246	246	246	246	246	246	246	246	246	246
	RmRf		**SMB**		**HML**		**WML**		**RMW**		**CMA**		**IML**	
Variable	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**	**Pak**	**India**
RmRf	1	1												
SMB	-0.1621	-0.2021	1	1										
HML	0.3364	0.1248	-0.1148	0.1748	1	1								
WML	-0.2448	0.17457	0.2823	0.2987	-0.4477	0.1657	1	1						
RMW	-0.2691	0.28454	0.5964	0.0273	-0.0867	0.3922	0.2733	0.29204	1	1				
CMA	-0.0354	0.27873	-0.6207	0.2498	-0.303	0.5582	-0.037	0.29673	-0.3974	0.29486	1	1		
IML	0.417	0.18805	0.0312	-0.0617	0.5723	0.2978	-0.2302	0.18989	-0.0722	0.19725	-0.2438	0.19326	1	1

Notes: Table 3 shows the descriptive statistics of the Pakistani and Indian stock markets with the help of a momentum and liquidity augmented 5FM. The constructed factors of market, size, value, momentum, with the average return of the PSX market risk premium being the lowest in comparison to the BSE. The table also states the standard deviation of each factor, as well as the minimum and maximum values.

negative association with the market risk-factor, but the liquidity factor appeared to have similar findings and statistically significant results at the 5% significance level (p-value = 0.0000) as PSX. The R-squared for BSE ranged from 50% to 69%. Ultimately, the liquidity factor had a significantly negative impact on the market risk-factor in PSX, though in BSE, it improved the market risk-factor as indicated by the findings.

Table 4
Optimal Lag Length Selection Criteria

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.526***	0.313***	0.313***	0.319***	0.386***	0.264***	RmRf	0.951***	0.980***	1.004***	1.024***	1.050***	1.020***
	-8.513	-6.424	-5.76	-7.197	-7.003	-4.881		-11.909	-12.454	-11.283	-9.945	-8.804	-14.538
Constant	-0.007	0.007**	-0.013***	0.007**	-0.013***	0.004	Constant	0.029***	0.036***	0.036***	0.041***	0.052***	0.016***
	(-1.527)	-2.004	(-3.163)	-2.239	(-3.239)	-1.043		-5.185	-6.462	-5.775	-5.543	-6.108	-3.221
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.238	0.151	0.125	0.183	0.175	0.093	R-squared	0.379	0.401	0.354	0.299	0.25	0.477

Notes: Table 4 displays the single-factor model (1FM) findings including coefficients and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

Table 5
Liu (2006) two-factor Model Results for Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.214***	0.024	-0.035	0.048**	0.083**	-0.026	RmRf	-0.376***	-0.567***	-0.634***	-0.745***	-0.898***	-0.244**
	-4.544	-0.846	(-1.443)	-2.038	-2.207	(-0.691)		(-2.751)	(-4.739)	(-4.439)	(-4.313)	(-4.362)	(-2.137)
IML	1.011***	0.936***	1.127***	0.875***	0.982***	0.942***	IML	1.161***	1.354***	1.433***	1.548***	1.704***	1.106***
	-15.894	-24.405	-34.652	-27.26	-19.427	-18.244		-11.031	-14.687	-13.025	-11.625	-10.748	-12.561
Constant	-0.011***	0.004*	-0.017***	0.004**	-0.017***	0.001	Constant	0.010*	0.013***	0.012**	0.014**	0.023***	-0.003
	(-3.377)	-1.962	(-10.406)	-2.554	(-6.671)	-0.268		-1.933	-2.997	-2.32	-2.247	-3.034	(-0.695)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.636	0.763	0.859	0.806	0.687	0.628	R-squared	0.594	0.69	0.628	0.558	0.5	0.689

Notes: Table 5 displays the two-factor model (Liu, 2006) findings including coefficients of market and liquidity factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

Table 6
Fama & French (1993) three-factor Model in Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.335***	0.224***	0.212***	0.203***	0.207***	0.197***	RmRf	1.076***	1.065***	1.073***	1.048***	1.093***	1.122***
	-5.943	-4.547	-4.894	-4.789	-4.584	-3.593		-23.161	-23.413	-21.365	-22.705	-22.905	-23.455
SMB	-0.265***	0.005	0.655***	-0.293***	0.163**	0.255***	SMB	0.600***	0.507***	0.514***	0.493***	0.614***	0.490***
	(-2.756)	-0.065	-8.869	(-4.039)	-2.113	-2.733		-11.597	-10.007	-9.194	-9.587	-11.556	-9.194
HML	0.889***	0.473***	0.851***	0.472***	1.027***	0.482***	HML	0.263***	0.367***	0.495***	0.764***	0.865***	0.216***
	-8.863	-5.392	-11.057	-6.249	-12.779	-4.949		-4.384	-6.255	-7.646	-12.834	-14.068	-3.506
Constant	0.014***	0.020***	0.015***	0.018***	0.015***	0.019***	Constant	0.002	0.009***	0.006*	0.003	0.008**	-0.007**
	-2.947	-4.688	-4.085	-4.785	-3.927	-4.066		-0.578	-2.919	-1.669	-1.028	-2.351	(-2.030)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.452	0.247	0.52	0.349	0.519	0.199	R-squared	0.836	0.844	0.84	0.89	0.901	0.81

Notes: Table 6 displays the 3FM findings including coefficients of market, size and value factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

An analysis using time-series OLS regression was conducted on the Fama and French (1993) three-factor model using PSX and BSE data, which is presented in Table 6. This analysis shows that the market-factor had a statistically significant at the 5% significance level (p-value = 0.0000), positive correlation with portfolio returns when the PSX dataset is used. The size-factor also had statistically significant coefficients at the 5% significance level (p-value = 0.0000), with the exception of the SM portfolio (0.005). The value-factor also had statistically significant at the 5% significance level (p-value = 0.0000), positive

results when the PSX dataset was used, and the R-squared values ranged from 20% to 52%. The BSE dataset yielded similar results; the market, size, and value factors all had statistically significant coefficients, and the R-squared values ranged from 81% to 90%. Overall, these findings indicate that the three-factor model significantly improved the results for both markets.

An examination of Carhart (1997) four-factor model is conducted using time-series OLS regression, which is reported in Table 7. The results were comparable to those of 3FM, but the momentum factor for PSX did not have statistically significant coefficients. Moreover, market and value factors are highly significant at the 5% significance level (p-value = 0.0000). The R-squared values for PSX ranged from 20% to 52%, which is similar to 3FM. On the other hand, BSE showed three statistically significant coefficients for the market risk factor. The size factor had all significant positive coefficients at the 5% significance level (p-value = 0.0000). However, the value factor had five significant coefficients at the 5% significance level (p-value = 0.0000) while one portfolio showed an insignificant coefficient (BH, 0.046). In contrast to PSX, the momentum factor for BSE had highly significant coefficients at the 5% significance level (p-value = 0.0000). Additionally, the R-squared values for BSE were from 80% to 86%, implying that the model was a good fit for the market.

Table 7
Carhart (1997) four-factor Model in Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.330***	0.229***	0.216***	0.203***	0.204***	0.201***	RmRf	0.113**	0.042	0.057	0.202***	-0.046	0.200***
	-5.835	-4.619	-4.973	-4.763	-4.497	-3.663		-1.973	-0.893	-0.887	-3.035	(-1.144)	-3.065
SMB	-0.237**	-0.018	0.634***	-0.293***	0.180**	0.231**	SMB	0.582***	0.488***	0.496***	0.478***	0.593***	0.473***
	(-2.387)	(-0.202)	-8.314	(-3.904)	-2.258	-2.396		-18.232	-18.524	-13.83	-12.922	-26.506	-13.01
HML	0.839***	0.513***	0.889***	0.472***	0.997***	0.524***	HML	-0.430***	-0.369***	-0.236***	0.155**	0.046	-0.447***
	-7.686	-5.373	-10.601	-5.722	-11.38	-4.941		(-8.361)	(-8.695)	(-4.088)	-2.597	-1.268	(-7.636)
WML	-0.089	0.073	0.067	0	-0.053	0.076	WML	0.872***	0.926***	0.919***	0.767***	1.031***	0.835***
	(-1.139)	-1.065	-1.129	-0.002	(-0.854)	-1.005		-19.355	-24.918	-18.203	-14.716	-32.678	-16.285
Constant	0.019***	0.017***	0.012**	0.018***	0.018***	0.016***	Constant	-0.008***	-0.001	-0.004*	-0.005**	-0.004**	-0.016***
	-3.032	-3.07	-2.553	-3.785	-3.632	-2.612		(-3.818)	(-0.644)	(-1.919)	(-2.206)	(-2.567)	(-6.863)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.455	0.25	0.522	0.349	0.521	0.202	R-squared	0.818	0.838	0.815	0.824	0.863	0.792

Notes: Table 7 displays the 4FM findings including coefficients of market, size, value and momentum factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

Table 8
Fama & French (2015) five-factor Model in Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.197***	0.203***	0.192***	0.209***	0.203***	0.183***	RmRf	0.280***	0.178***	0.104***	0.182***	0.275***	0.048
	-4.9	-4.17	-4.463	-4.932	-4.795	-3.754		-4.26	-3.373	-3.078	-3.104	-4.154	-1.085
SMB	0.574***	0.429***	0.468***	-0.043	0.680***	1.031***	SMB	0.639***	0.562***	0.696***	0.533***	0.668***	0.298***
	-5.611	-3.466	-4.286	(-0.403)	-6.329	-8.327		-15.091	-16.579	-31.919	-14.109	-15.634	-10.377
HML	0.996***	0.643***	0.706***	0.616***	1.286***	0.854***	HML	-0.051	0.018	0.126***	0.423***	0.545***	-0.229***
	-12.716	-6.789	-8.44	-7.464	-15.638	-9.006		(-1.050)	-0.476	-5.039	-9.769	-11.137	(-6.958)
RMW	-1.663***	-0.304**	-0.191*	0.024	-0.135	-0.280**	RMW	0.644***	0.772***	1.395***	0.692***	0.724***	-0.236***
	(-15.524)	(-2.345)	(-1.673)	-0.212	(-1.197)	(-2.159)		-6.982	-10.453	-29.351	-8.403	-7.783	(-3.779)
CMA	0.137	0.429***	-0.408***	0.388***	0.682***	0.971***	CMA	0.089	0.046	-0.493***	0.106	0.029	1.205***
	-1.434	-3.706	(-3.992)	-3.853	-6.792	-8.39		-0.895	-0.576	(-9.615)	-1.189	-0.292	-17.871
Constant	0.015***	0.019***	0.017***	0.016***	0.013***	0.016***	Constant	-0.004*	0.002	-0.004***	-0.003*	0.001	-0.011***
	-4.378	-4.52	-4.636	-4.498	-3.624	-3.861		(-1.927)	-1.012	(-3.571)	(-1.727)	-0.421	(-6.970)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.737	0.309	0.555	0.389	0.604	0.403	R-squared	0.798	0.828	0.862	0.836	0.835	0.839

Notes: Table 8 displays the 5FM findings including coefficients of market, size, value, profitability and investment factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

The results of Table 8, which used an OLS regression technique and the PSX and BSE datasets, indicate that the market and value factors for PSX have statistically significant coefficients at the 5% significance level (p-value = 0.0000), while the size and investment factors have five significant coefficients at the 5% significance level (p-value = 0.0000), although one portfolio each is insignificant (BH, -0.043) and (SH, 0.137) respectively. The profitability factor for the PSX market has four statistically significant coefficients and two insignificant coefficients. On the other hand, the BSE market factor has five statistically significant coefficients and one insignificant (BL, 0.048). Furthermore, all size and profitability factors for the BSE market have significant coefficients at the 5% significance level (p-value = 0.0000), while the value factor has four significant coefficients and the investment factor has two in the results. In addition, the R-squared ranges from 31% to 74% for PSX and 80% to 86% for BSE.

Table 9
Liquidity augmented Fama & French (2015) five-factor Model in Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.025	0.009	0	0.017*	0.051*	-0.006	RmRf	0.136**	0.04	0.04	0.136**	0.04	0.039
	-1.53	-0.351	-0.003	-1.655	-1.771	(-0.235)		-2.416	-1.291	-1.291	-2.416	-1.291	-0.916
SMB	0.387***	0.200***	0.249***	-0.249***	0.525***	0.805***	SMB	0.636***	0.723***	0.723***	0.636***	0.723***	0.212***
	-9.508	-3.344	-9.987	(-9.929)	-7.474	-12.508		-16.162	-33.634	-33.634	-16.162	-33.634	-7.203
HML	0.406***	0.011	0.070***	-0.051**	0.742***	0.246***	HML	-0.465***	0.005	0.005	0.535***	0.005	-0.460***
	-10.886	-0.205	-3.076	(-2.222)	-11.517	-4.175		(-7.057)	-0.142	-0.142	-8.115	-0.142	(-9.346)
RMW	-1.574***	-0.221***	-0.104***	0.127***	-0.045	-0.203***	RMW	0.419***	0.240***	1.240***	0.419***	0.240***	-0.085
	(-37.055)	(-3.540)	(-3.989)	-4.843	(-0.613)	(-3.020)		-4.715	-4.949	-25.578	-4.715	-4.949	(-1.280)
CMA	0.147***	0.443***	-0.395***	0.398***	0.688***	0.986***	CMA	-0.281***	-0.730***	-0.730***	-0.281***	-0.730***	1.403***
	-3.904	-8.026	(-17.178)	-17.183	-10.602	-16.593		(-2.674)	(-12.742)	(-12.742)	(-2.674)	(-12.742)	-17.904
IML	0.891***	1.024***	1.004***	0.993***	0.777***	0.998***	IML	-0.218	1.129***	0.129*	0.782***	0.129*	-0.721***
	-35.37	-27.651	-65.205	-63.939	-17.874	-25.062		(-1.523)	-14.453	-1.656	-5.465	-1.656	(-6.754)
WML	-0.009	0.068**	0.039***	-0.027**	-0.058	0.082**	WML	0.933***	0.322***	0.322***	-0.067	1.322***	0.366***
	(-0.425)	-2.153	-3.008	(-2.019)	(-1.558)	-2.402		-9.283	-5.858	-5.858	(-0.667)	-24.08	-4.884
Constant	-0.005***	-0.008***	-0.008***	-0.005***	-0.002	-0.011***	Constant	-0.008***	-0.007***	-0.007***	-0.008***	-0.007***	-0.008***
	(-2.818)	(-3.079)	(-7.353)	(-4.808)	(-0.660)	(-3.855)		(-3.900)	(-6.006)	(-6.006)	(-3.900)	(-6.006)	(-5.499)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.84	0.724	0.858	0.848	0.716	0.724	R-squared	0.824	0.863	0.866	0.842	0.871	0.846

Notes: Table 9 displays the 6FM findings including coefficients of market, size, value, profitability, investment and liquidity factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

Table 10
Liquidity and Momentum augmented Fama & French (2015) five-factor Model in Pakistan and India

PSX	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)	BSE	(SH)	(SM)	(SL)	(BH)	(BM)	(BL)
Factor	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**	**Factor**	**PF1**	**PF2**	**PF3**	**PF4**	**PF5**	**PF6**
RmRf	0.025	0.009	0.000	0.017*	0.051*	-0.006	RmRf	0.136**	0.040	0.040	0.136**	0.040	0.039
	-1.53	-0.351	-0.003	-1.655	-1.771	(-0.235)		-2.416	-1.291	-1.291	-2.416	-1.291	-0.916
SMB	0.387***	0.200***	0.249***	-0.249***	0.525***	0.805***	SMB	0.636***	0.723***	0.723***	0.636***	0.723***	0.212***
	-9.508	-3.344	-9.987	(-9.929)	-7.474	-12.508		-16.162	-33.634	-33.634	-16.162	-33.634	-7.203
HML	0.406***	0.011	0.070***	-0.051**	0.742***	0.246***	HML	-0.465***	0.005	0.005	0.535***	0.005	-0.460***
	-10.886	-0.205	-3.076	(-2.222)	-11.517	-4.175		(-7.057)	-0.142	-0.142	-8.115	-0.142	(-9.346)
RMW	-1.574***	-0.221***	-0.104***	0.127***	-0.045	-0.203***	RMW	0.419***	0.240***	1.240***	0.419***	0.240***	-0.085
	(-37.055)	(-3.540)	(-3.989)	-4.843	(-0.613)	(-3.020)		-4.715	-4.949	-25.578	-4.715	-4.949	(-1.280)
CMA	0.147***	0.443***	-0.395***	0.398***	0.688***	0.986***	CMA	-0.281***	-0.730***	-0.730***	-0.281***	-0.730***	1.403***
	-3.904	-8.026	(-17.178)	-17.183	-10.602	-16.593		(-2.674)	(-12.742)	(-12.742)	(-2.674)	(-12.742)	-17.904
IML	0.891***	1.024***	1.004***	0.993***	0.777***	0.998***	IML	-0.218	1.129***	0.129*	0.782***	0.129*	-0.721***
	-35.37	-27.651	-65.205	-63.939	-17.874	-25.062		(-1.523)	-14.453	-1.656	-5.465	-1.656	(-6.754)
WML	-0.009	0.068**	0.039***	-0.027**	-0.058	0.082**	WML	0.933***	0.322***	0.322***	-0.067	1.322***	0.366***
	(-0.425)	-2.153	-3.008	(-2.019)	(-1.558)	-2.402		-9.283	-5.858	-5.858	(-0.667)	-24.08	-4.884
Constant	-0.005***	-0.008***	-0.008***	-0.005***	-0.002	-0.011***	Constant	-0.008***	-0.007***	-0.007***	-0.008***	-0.007***	-0.008***
	(-2.818)	(-3.079)	(-7.353)	(-4.808)	(-0.660)	(-3.855)		(-3.900)	(-6.006)	(-6.006)	(-3.900)	(-6.006)	(-5.499)
Obs.	246	246	246	246	246	246	Obs.	246	246	246	246	246	246
R-squared	0.84	0.724	0.858	0.848	0.716	0.724	R-squared	0.824	0.863	0.866	0.842	0.871	0.846

Notes: Table 10 displays the 7FM findings including coefficients of market, size, value, profitability, investment, liquidity and momentum factor and associated t-statistics in parentheses with p-value as *** p<0.01, ** p<0.05, * p<0.1.

The results of applying liquidity augmented 5FM on PSX and BSE datasets are pre-

sented in Table 9. This reveals that when liquidity is added to the 5FM, the market factor yields weak and insignificant results, with only two coefficients displaying weakly significant results for the PSX market. Beside, size, investment and liquidity factors, however, show statistically significant coefficients at the 5% significance level (p-value = 0.0000) for all portfolios in the PSX market. However, the value and profitability factors show four and five statistically significant coefficients respectively, and the R-squared range from 71% to 86% for PSX. As for the BSE market, the results show highly significant results, as the market show significant coefficient while size and liquidity factors all have statistically significant coefficients at the 5% significance level (p-value = 0.0000) for all portfolios. Moreover, the value, profitability and investment factors have five statistically significant coefficients for the BSE market, and the R-squared range from 80% to 86%.

Table 10 illustrates the liquidity and momentum augmented 5FM using both the PSX and BSE datasets. In the PSX results, the market factor showed insignificant coefficients for four portfolios, while two portfolios showed weak significant coefficients, indicating no effect of the 1FM. However, the size, investment, and liquidity factors all displayed statistically significant coefficients across all portfolios at the 5% significance level (p-value = 0.0000), while the value and profitability factors had five significant coefficients. Additionally, the momentum factor showed four portfolios with significant coefficients. The R-squared values ranged from 72% to 86%. The BSE results showed slightly different, but similar results. The market factor displayed two significant coefficients, again indicating no effect of the 1FM. The size and investment factors had significant coefficients across all portfolios at the 5% significance level (p-value = 0.0000), while the profitability, liquidity, and momentum factors had five significant coefficients at the 5% significance level (p-value = 0.0000) for the market. The value factor showed three significant coefficients for the market. The corresponding R-squared values ranged from 82% to 86%.

Robustness of Factors in the Models

This study investigates the liquidity and momentum augmented various APMs using time-series OLS regression technique by following Fama and French (1993) in both BSE and PSX market. After analyzing all the models, the following conclusive results are derived which are presented in Table 11:

Table 11
Conclusive findings at a Glance

Model	1FM		2FM		3FM		4FM		5FM		6FM		7FM	
Factor	**Pak**	**Ind**	**Pak**	**Ind**	**Pak**	**Ind**	**Pak**	**Ind**	**Pak**	**Ind**	**Pak**	**Ind**	**Pak**	**Ind**
RmRf	6	6	3	6	6	6	6	3	6	5	2	6	2	2
IML	-	-	6	6	-	-	-	-	-	-	6	6	6	5
SMB	-	-	-	-	5	6	5	6	5	6	6	6	6	6
HML	-	-	-	-	6	6	6	5	6	4	4	5	5	3
WML	-	-	-	-	-	-	0	6	-	-	-	-	4	5
RMW	-	-	-	-	-	-	-	-	4	6	5	5	5	5
CMA	-	-	-	-	-	-	-	-	5	2	6	5	6	6

Table 11 displays the conclusive findings at a glance includes risk-factors, significant coefficients out of total (6 portfolios) for all models and both countries accordingly.

The significant coefficients out of six-portfolios based on both countries are presented in the Table 11. The findings reveal that market-factor performs better findings but with the augmenting of liquidity, it shows insignificant behaviour in the case of PSX (6FM and 7FM). However, by augmenting momentum factor in the specification, BSE behave insignificant findings for market-factor (4FM and 7FM). Moreover, the liquidity-factor performs better findings for both countries indicating that liquidity contributes significant impact on portfolio returns. Similarly, size-factor performs better findings for both markets using all models. Additionally, the value-factor demonstrates significant results and is not redundant for both markets with the exception for BSE (7FM). Furthermore, momentum factor shows statistically significant results for BSE though insignificant findings for PSX (4FM). Besides, the profitability-factor also demonstrates significant impact on explaining momentum of portfolio returns for both markets. In addition, the investment-factor demonstrates significant findings for PSX while in the case of BSE, it shows insignificant findings (5FM) though by augmenting liquidity, it substantially improves the findings and diverts into significant outcomes (6FM and 7FM).

Model Performance Tests (GRS F-test)

The GRS F-test results for each model are reported in Table 12. The GRS F-test revealed that none of the alpha coefficients were equal to zero (α=0), which means the null hypothesis of all alpha coefficients being equal to zero (α=0) was rejected and the alternative hypothesis of all alpha coefficients not being equal to zero ($\alpha \neq 0$) was accepted. According to the absolute mean alpha (AMA) from the GRS F-test, the 4FM and LM5FM are the best models for pricing assets at the Indian Stock Exchange (BSE) and Pakistan Stock Exchange (PSX) respectively.

Table 12
Gibbons, Ross and Shanken (1989) F-test for BSE and PSX

Model	Abs. Mean Alpha (BSE)	Abs. Mean Alpha (PSX)	Mean Adj.-R2 (BSE)	Mean Adj.-R2 (PSX)	GRS F-Test (BSE)	GRS F-Test (PSX)	GRS-P-value (BSE)	GRS-P-value (PSX)
1FM	0.040382	0.004109	0.355592	0.151829	13.858212	16.869379	0.000	0.000
LCAPM	0.016682	0.000422	0.605749	0.725705	7.020333	16.49561	0.000	0.000
3FM	0.008766	0.02335	0.851739	0.366998	7.524448	7.643329	0.000	0.000
4FM	0.000872	0.023092	0.943943	0.366565	5.417019	5.480506	0.000	0.000
5FM	0.002021	0.022314	0.951687	0.484708	6.252906	7.097692	0.000	0.000
L5FM	0.001728	0.00055	0.962238	0.901427	0.756779	0.200081	0.604	0.976
LM5FM	0.001936	0.000101	0.971166	0.902686	0.739055	1.069003	0.000	0.382

Notes: Table 12 presents the results from the GRS F-test for the purpose of verifying the robustness of the results. The GRS F-test, P-value absolute Mean Alpha and mean adjusted R-2 illustrate the results for BSE and PSX.

Conclusion and Implications

Conclusively, the findings determine that market performs significant contribution independently though augmenting the liquidity-factor changes the impact badly (from significant to insignificant) in PSX while on the other side, investment-factor shows insignificant contribution independently though augmenting the liquidity-factor changes the impact positively (from insignificant to significant) in BSE. Moreover, the value-factor is not re-

dundant for both the markets. On the other hand, based on absolute mean alpha (AMA) of GRS test, the Carhart (1997) four-factor model (4FM) for Indian Stock Exchange (BSE) and liquidity and momentum adjusted 5FM for Pakistan Stock Exchange (PSX) outperform (7FM) other asset pricing models according. To conduct future research in this discipline, the researchers can use leverage risk-factor as used by Azam and Ilyas (2011) which produced statistically significant findings for PSX market.

The findings determine that market-factor performs significant contribution independently though augmenting the liquidity-factor which resultantly enhances an insignificant and tenuous impact (from significant to insignificant) in PSX while on the other side; investment-factor shows insignificant contribution independently though augmenting the liquidity-factor which resultantly enhances the impact positively (from insignificant to significant) in BSE. Moreover, the value-factor is not redundant for both the markets. On the other hand, based on absolute mean alpha of GRS test, the Carhart (1997) four-factor model for Indian Stock Exchange and liquidity and momentum adjusted 5FM for Pakistan Stock Exchange (Azam, 2021) seven-factor model outperform other asset pricing models. According to the absolute mean alpha from the GRS F-test, the 4FM and LM5FM are the best models for pricing assets at the Indian Stock Exchange and Pakistan Stock Exchange respectively.

The research was carried out in the equity markets of India and Pakistan. As a result, the study's conclusions may not be generalizable to other equity markets; nonetheless, future research should take into account other nations and equities markets. Second, multidimensional liquidity has a substantial impact on explaining portfolio returns; future study should take into account various other factors such as leverage, downside risk, and maximum drawdown in such a nexus to examine further market robustness. The study on the significant influence of multidimensional liquidity and momentum using nested multifactor asset pricing models might be worthwhile for investors and portfolio managers to consider before designing diversified portfolios in both markets. Second, the implications of these asset pricing strategies can significantly improve results by beating the market utilizing macroeconomic drivers as market intrinsic factors. The findings are likely to be useful to policymakers. To the best of the authors' knowledge, there has been no empirical study on comparison studies employing both stock markets datasets, particularly for Liu's multidimensional liquidity and momentum enhanced numerous nested asset pricing models initiatives to formulate more robust findings. In terms of constructing efficient portfolios to increase investors' returns and yield excess returns in both markets, the study's findings may have significant ramifications for stock investors and portfolio managers. On the basis of such comprehension, the markets may permit the reformation of outdated policies to encourage investors' capabilities in sustainable growth in the new economic contexts. The study's conclusions may have a big impact on APMs in India and Pakistan.

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Appendix

Table
Appendix: Factor Spanning Test Regression results for Pakistan

Pakistan	-1 RmRf	-2 SMB	-3 HML	-4 WML	-5 RMW	-6 CMA	-7 IML
SMB	-0.08753		-0.32050***	0.16719	0.39701***	-0.63095***	0.20616*
	-0.16287		-0.06922	-0.12535	-0.05787	-0.05846	-0.10642
HML	0.14455	-0.26927***		-0.61319***	0.07192	-0.37612***	0.67558***
	-0.14907	-0.05815		-0.10792	-0.05809	-0.061	-0.08753
WML	-0.07993	0.04651	-0.20304***		0.06365*	-0.01478	0.0333
	-0.0858	-0.03487	-0.03574		-0.03327	-0.03792	-0.05655
RMW	-0.48035***	0.43259***	0.09328	0.24930*		-0.07398	-0.10528
	-0.16711	-0.06305	-0.07533	-0.13031		-0.0749	-0.11178
CMA	-0.08292	-0.53752***	-0.38139***	-0.04526	-0.05784		-0.0097
	-0.15032	-0.0498	-0.06186	-0.11611	-0.05856		-0.09903
IML	0.44763***	0.07889*	0.30771***	0.04581	-0.03697	-0.00436	
	-0.09634	-0.04072	-0.03987	-0.07778	-0.03926	-0.04448	
RmRf		-0.01452	0.02854	-0.04765	-0.07312***	-0.01614	0.19402***
		-0.02701	-0.02943	-0.05115	-0.02544	-0.02927	-0.04176
_cons	0.01015	-0.00665**	-0.01061***	0.04715***	-0.00138	0.00446	0.02131***
	-0.00708	-0.00286	-0.00308	-0.00452	-0.00277	-0.00313	-0.00447
Obs.	246	246	246	246	246	246	246
R-2	0.24685	0.5957	0.52394	0.27273	0.40246	0.52999	0.40417

India	-1 RmRf	-2 SMB	-3 HML	-4 WML	-5 RMW	-6 CMA	-7 IML
SMB	-0.16708***		0.03474	0.07733***	-0.06997**	0.21414***	-0.12304***
	-0.04498		-0.03956	-0.02548	-0.02905	-0.02042	-0.01632
HML	-0.30098***	0.09747		0.50373***	0.05714	0.10088**	-0.27055***
	-0.07497	-0.11098		-0.02788	-0.04914	-0.04113	-0.02474
WML	0.35999***	0.50433***	1.17108***		0.04041	0.0888	0.38939***
	-0.11587	-0.16617	-0.06481		-0.07509	-0.06327	-0.03879
RMW	-0.01721	-0.35602**	0.10363	0.03153		0.22658***	0.24906***
	-0.10449	-0.14784	-0.08912	-0.05859		-0.05407	-0.03771
CMA	0.30694**	1.52376***	0.25588**	0.09688	0.31685***		0.33540***
	-0.12188	-0.14532	-0.10433	-0.06903	-0.07562		-0.0433
IML	0.04283	-1.62685***	-1.27519***	0.78943***	0.64718***	0.62324***	
	-0.16842	-0.21585	-0.11662	-0.07865	-0.09799	-0.08047	
RmRf		-0.34299***	-0.22025***	0.11331***	-0.00694	0.08855**	0.00665
		-0.09233	-0.05486	-0.03647	-0.04215	-0.03516	-0.02615
_cons	-0.00412*	0.01535***	0.00155	0.00039	-0.00109	-0.00534***	0.00401***
	-0.00235	-0.00324	-0.00202	-0.00133	-0.0015	-0.00122	-0.00089
Obs.	246	246	246	246	246	246	246
R-2	0.6144	0.58912	0.69656	0.77792	0.76069	0.77397	0.7829

Standard errors are in parenthesis with *** p<0.01, ** p<0.05, * p<0.1